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# A Theoretical Approach to Curricula

*Summary:* This paper was first presented at the International Seminar on *Comparative Studies of Mathematical Curricula in Different Countries,* held in Frascati (Italy) in may 1987. Though it centers on some definite questions – essentially, "What is a curriculum?", and "How does a curriculum come into existence?" – it is intended as an introduction to the theory of the didactic transposition of knowledge, in whose setting it tries to make sense of the aforementioned, central problems of didactic theory. Due to lack of space, the chief theoretical notions on which it hinges – notably those of demands (as opposed to needs), of conditions and constraints, etc. – will not be formally introduced to the reader. Much to the contrary, it has been the author's intent to make their proper meaning grow out of their straightforward use throughout the text. The argument expounded often wanders from admitted truth and the general style of analysis is bound to come as a surprise to the casual reader, a fact for which the author can, unfortunately, offer no apologies, but which he would rather see as an indication that something is indeed being communicated.

## 1. On the question "What is a curriculum?"

In trying to theorise about curricula and the social activity of curriculum development, the first question one must raise is: "What is a curriculum?" However unassuming it may look at first, this question deserves careful consideration, for it leads directly into the vortex of didactic theory.

What must be cautiously appraised is, indeed, the *kind of answer* didactic theory should supply. The kind of answer we should acknowledge in the first place is of the type readily available to the layman, namely, *dictionary definitions* (as for example: "A course of study") .Such an answer contrasts in varying degrees with those obtainable from "clerics" (as opposed to "laymen"), i.e. from experts or "professionnals" in the field, of which the following, voluntarily anonymous quote seems to be representative: "In recent years curriculum has increasingly been defined as a selection from the culture of a society; and the curriculum is planned by a process of cultural analysis."

Now, in order to account for the distinction between dictionary and expert definitions, i.e. between the culture of "laymen" and the culture of "clerics", some theoretical provisions must be made. In this respect, let me first distinguish as a central character the *society*, i.e. the "social formation" within which the whole story will take place – such as, for example, present-day French society. In the given society, I now mark off a particular subsystem which I style the *teaching system*.

On the basis of the elemental distinction between society and its teaching system, dictionary definitions can now be ascribed to society (and its "laymen"), as opposed to the teaching system and its experts. More precisely, dictionary definitions are constituents of yet another entity, *culture*, to be defined as the complex totality of representations which embody a common experience of social life, are shared in varying degrees by almost every member of the society in question, and force upon those who share them a common sense of identity [1].

### *1.2. The noosphere*

Next, what about professionnals' definitions? This question calls for yet another theoretical notion, whose status deserves to be carefully examined.

[1] It should be emphasized here that the meaning ascribed to the word "culture" in the present context result from the rôle assigned to it within the theoretical framework under consideration. The word should thus be stripped of the many connotations with which culture (in our sense) has loaded it. See WILLIAMS 1983.

The teaching system is not a thing in one piece. It does not consist only of teachers and students, texbooks, homework assignments, and so forth. Like any social institution, *it has to attend to the maintenance of its relations with society as a whole*. Accordingly, a part of it will specialise in the overseeing of the relationship between the teaching system proper and its societal environment. This is a quite general requirement of social life, which no institution can elude.

Now a very distinctive feature of such institutional offshoots is that they usually go *unlabelled*, and therefore, up to a point, unnoticed. In other words, they usually remain unknown to the society's culture – so much so that no record of them is available in language. For these unnamed realities of social life I therefore had to supply a generic name, and I chose to style them *noospheres* [2].

Essentially, the *didactic* noosphere is thus the "sphere" of those who "think" about teaching. Crudely put, it consists of all those persons who share an interest in the teaching system, and who "act out" their impulses in some way or another. Most noospherians, by the way, are only part-time militants, be they teachers, mathematicians, or what have you. It should also be clear that different people take up different *positions* in the noosphere. Some of them are curriculum-designers while others choose to be didactic theorists. The noosphere thus appears to be the union of a host of microworlds.

Let us turn now more concretely to the rôle played by the curriculum-developer within the noosphere. In going about his business, the curriculum-developer performs a function which happens to be central to the activity of the noosphere and which I shall abstractly describe as one of *negotiation with society*. For the noosphere, in effect, has to fulfil the function of a "shock absorber" with respect to the threats launched by society against its teaching system. More precisely, it is the task of the noosphere to cope with the *demands* made by society on the teaching system, by transmuting them into conditions acceptable to both parties – society and its teaching system. Obviously, this is a job of paramount necessity.

In the case of the teaching system, this function is fulfilled in a very conspicuous way. Indeed, unless rapidly muted, any demand, from any segment of society, will tend to be translated *in terms of knowledge-to-be-taught*. (This fact should remind us that the teaching system is primarily concerned, as a social institution, *with the social processing* of *knowledge*.) Not unexpectedly, then, curriculum-developers play a significant rôle in this very complex transmutation.

Let us a take a new step forward. Insofar as they engage in action within the noosphere, curriculum-developers will answer the question "What is a curriculum?" in such a way that their answer will be useful to them as strategic weapons in the negotiation process. For this reason, such answers are sure to embody definite values and views of the didactic world, i.e. a spontaneous philosophy or "theory" of the teaching system and of its relations to society, as well as policy principles of change and adaptation to society.

To quote a case in point, let me mention here the seemingly innocent use of the word "need" by English-speaking writers on curriculum-development – most of whom raise the question of the relationship of curricula to so-called "social needs". How can they know about the needs of society? I can know that I *want* sugar in my coffee; I can *ask* for more sugar – which is a *demand*. But whether I really *need* sugar is quite another story. What the curriculum-developer, like any receptive member of society, is faced with is the *demands* of society, expressed in more or less systematic form. What the objective *needs* of society are remains open to question – especially as *prospective* needs are supposedly aimed at.

Now the unassuming use of the word "need" tends to imply that curriculum-developers both know what the society's needs are and will be, and duly seek to cater for them. This appears

[2] A word flippantly borrowed from Teilhard de Chardin's *The Phenomenon of Man*, originally coined from the Greek word *noos, noûs*, mind.

to be quite fair as a (most certainly unconscious) trick in the ongoing negotiation with society. But didactic analysis should not take it at its face value. Let me add that it is the duty of the didactician to try to "deconstruct" it, if only to avert the "sanctified fallacies" of the noospheric ideology – e.g. on the social utility of mathematics.

## 2. On a potential didactic answer

### *2.1. The transparency illusion*

A major characteristic is shared by both the members of the teaching system – including noospherians – and society itself – i.e. the "man in the street". The ordinary view of the social world bestows upon it the character of a self-evident, unquestionable and compelling reality whose existence and features (as identified by culture) are taken for granted. The "taken-for-grantedness" of social reality is the ontological foundation of everyday life: to engage in doubt about it would amount to challenging our familiarity with it and, up to a point, our own sense of identity. Conversely, to give in to it boils down to returning to our familiar world, in which things are as they should be and look as they always did. It is precisely this sense of immediacy which I style *the transparency illusion*. The delusive feeling that the world is not to be questioned is imposed on us by everyday life education. But it is a major historical import of, first philosophy and then science, that the world can indeed be questioned. I may take for granted that $\sqrt{x}$ is not a polynomial function (on the real numbers), and it is obviously better to do so in routine mathematical activity; but mathematics can also *prove* that this is indeed so. In proving that this function is not a polynomial function, I make a transition from a (locally) "preconstructed" world to a (locally) "constructed" world. (In fact, almost all the so-called "crises" with which mathematics has historically been confronted – What is a number? What is continuity? What is a set? – can be thought of in just these terms.)

Preconstruction and taken-for-grantedness pervade our usual relation to the world. Usually, what is taken for granted is the world as it is *now*. In the case of the teaching system, however, another kind of immediacy must be posited in order to achieve a better understanding of the way people relate to it. For what is also taken for granted in this case is the supposedly easygoing relationship *between decisions and change* –between *free will*, or even simply "good will", and *facts*. In more concrete words, we are usually tempted into believing that it is enough for us to make decisions – and the world will obey them.

This legalist creed is generally not even challenged by the bitter disappointments that naive decision-making is sure to generate. For its origin is deep-rooted in everyday life experience. (If I decide to raise my arm or close my eyes, this will certainly happen.) However, it is certainly no easy matter to elucidate the reasons why, in the case of the teaching system, such an attitude towards decision and change tends to persist, whereas, for example, we no longer think that, in order to reduce unemployment, we must simply decide to do so. Be this as it may, any scientific view of the world should dispute the validity of this spontaneous "theory" of social reality.

### *2.2. Towards a didactic theory of curricula*

A didactic-theoretical approach to the curriculum concept should not be expected to reflect faithfully the various ways in which people use the word. On the one hand, it must lead to a "constructed notion" – as opposed to the preconstructed notions supplied by empirical observation. On the other hand, it should pave the way for a theoretical explanation of the way people "handle" curricula, make sense of them, and engage in symbolic manipulation of them.

Such a theoretical endeavour is immediately confronted with the old pitfall of empiricism: how can one account, in a unified theoretical manner, for an empirically diverse reality? In the particular perspective of comparative studies, the general empiricist inclination towards the diverse may be encouraged by explicit interest in comparisons, insofar as empirical distinctions, not the general laws governing them, are more or less unconsciously sought after. This might even condense into the belief that a circumstancial taxonomy of curricula across the globe is the unique possible form of a *theory* of curricula – which of course it cannot be.
Leaving aside this – scientifically absurd – precept, I shall hereafter ignore the fascinating variety of concrete cases to which the word "curriculum" is applied or tends to be applied, and simply look for a feature common to all and worthy of being accounted for in a didactic theory. I believe this is possible, provided we look at the whole gamut of concrete cases in a definite didactic perspective, that of the formation of *taught knowledge*, i.e. the knowledge which becomes visible, so to speak, in the classroom.
In doing so, I am fully aware that I do violence to a most cherished tenet of many noosperians, for whom prescription of educational goals should take precedence over the definition of subject-matter contents. But this is in keeping with a central, distinctive feature of the style of theorisation I am aiming at: the postulate that the didactic world "revolves round knowledge" and that any didactic fact whatsoever must in the last instance be explained by the way *knowledge is engaged in its production* – a principle to which the word "didactic" bears witness. Let me add in passing that didactic theory does not seek to evade the duty of accounting for those facts which, while pertaining to the didactic sphere, seem to make the question of knowledge irrelevant; a "knowledge-centered" teaching system is neither more nor less amenable to didactic theory than, for instance, a so-called "child-centered" or "goal-directed" one. Consequently, it must be clear that we shall not take the way people – laymen as well as "professionnals" – experience and label the world and its various segments at its face value, but try to explain it within our didactic theoretical framework.

### *2.3. Early steps*

To do so, I start from the most neutral definition of "curriculum" that I can conceive of, and first define the word "curriculum" to refer to *a course of study*. How can I now make sense of this – heavily empirical – definition within my theoretical framework? Simply by subsuming it under a concept which is central to didactic theory: that of the *didactic order*. To put it plainly, I now define, a curriculum to be a determined arrangement or *state* of a teaching system.
Up to now, then, I simply refer to curricula as "things" which are just "out there". Let me further raise yet another question: how have these (social) realities been brought about? In asking this question, we are drawing nearer to the question of curriculum-development. Any answer to it will depend heavily on the conception of social change which our theory embodies. To make things clearer, I shall rely here on a three-fold typology usually ascribed to the German sociologist, Max Weber.
In the first place, change may be carried out through the action of a charismatic leader. (We should think here of men like Dienes, Papy, or even Piaget.) Charismatic leaders usually proffer broad revolutionary and innovative views directed against the traditional state of affairs [3]. But, as Weber rightly observed, the lot of charisma is that it never lasts: charisma is always *in statu nascendi*, for it is rapidly routinised.

[3] A rule to which John Dewey appears to be a noticeable exception. In his *Experience and Education* (1938), he wisely remarks: "There is always the danger in a new movement that in rejecting the aims and methods of that which it would supplant, it may develop its principles negatively rather than positively and constructively. Then it takes its clew in practice from that which is rejected instead of from the constructive development of its own

This leads to a second kind of change. Charisma is traditionalised, i.e. the commotion it may have created dies down while its consequences are taken care of by faceless bureaucrats. Lastly, charisma may give way to a third kind of change, which I shall style *rational change*.
I am now in a position to tackle the question of the formation of curricula. All three kinds of change should generally be invoked: concrete curricula are the outcomes of a mixture of factors. In contrast, "curriculum-development" can be regarded as both a doctrine and a social practice *imbued with rational ideals*. While, to a certain extent, both the traditional view and the charismatic exaltation acknowledge the right of social change to a degree of opacity, the rational venture presupposes transparency – of both the process and its outcome.

## 3. Didactic transposition theory

### *3.1. The rational-bureaucratic venture*

The word "rational" can be misleading in this context. For it points to what can aptly be called a rational-bureaucratic view of the world, as opposed to a rational-scientific approach. Broadly speaking, it relates to the process of rationalisation which, according to Weber, is a major feature of modern technological societies. In this respect, the curriculum-development movement appears to be a straighforward response of the teaching system to the growing rationalisation of society. In essence, any rational-bureaucratic *Weltanschauung* indulges in the transparency illusion. It dismisses the indisputable fact that the social world, although partly created by man, does not obey him – that, so to speak, it is his creation, not his creature. Curriculum-development is no exception to the rule: there is always a far cry from its intended outcomes to its objectively observable effects.
What a didactic theory of curricula and curriculum-development should be interested in, however, is the reasons why such a failure is regularly reported, to the extent that trying and failing and trying hard again and waiting for the next fiasco seems to be the stuff that curriculum-development is made of. This state of affairs raises a big question – the foundation question of scientific understanding.

### *3.2. The ecological problematic*

If we want to arrive at a scientific view of the rationale of curriculum-development, we have first to posit a very exacting metatheoretical assumption. To wit, that social reality *is* subject to *laws* or *law-like determinations*. As this statement may very clearly provide food for controversy, I shall make a brief comment and even give an example.
The comment first. The assumption of determinism is not to be thought of in terms of cause and effect, but rather in terms of *conditions* and *constraints*. (What conditions and constraints are will soon become clearer.) In this perspective, the question to put in the face of a given fact will no longer be, "What is the cause of it?", but rather, "Why is it as it is, and not otherwise?" In changing the question, we make a transition from the old cause-effect paradigm to a more open and suitable problematic which I have come to call the *ecological* problematic. Ecological analysis can indeed be summed up in two closely connected questions. Firstly, what are the *conditions* under which the given fact becomes, so to speak, viable? Secondly, what are the *constraints* which, in the given situation, might prevent these conditions from being satisfied? I shall use just these notions in presenting the following example.
While observing the French lower secondary education school system, my attention was drawn, years ago, to a very particular fact which cannot fail to appear – mathematically

philosophy." (DEWEY 1938, p.2).

speaking – as something of an oddity. It seemed in fact that mathematics teachers tended to distinguish between, let us say, the "natural" number 3 and the "signed" number, +3. This state of affairs could certainly be described, from a mathematical point of view, by saying that, in passing from the set of natural numbers **N** to the set of integers **Z**, teachers failed to eventually embed **N** into **Z**. But such a *description* could in no way be regarded as a valuable *explanation*. The failure to identify natural integers with positive integers could no more be explained by a supposed lack of mathematical "expertise" on the part of these teachers. Moreover, it was quite clear that, as persons –not teachers –, they made no real difference between, let us say, 3 and +3. So that, indeed, there should have been many very good reasons why this particular phenomenon *should not have occurred*.

It so happened that I stumbled on – rather than discovered – what I now believe to be the ecological explanation of this oddity, I mean the *particular constraint* which inhibited the occurrence of the identification of 3 with +3. In fact, I realised that, in teaching algebraic sums, these teachers indulged in yet another amazingly wide-spread mathematical oddity – which in fact appeared at the time to be usual practice in the French *collèges*. In the calculation of, for example, what they would write initially

$$(+7) – (–3) + (–5)$$

teachers usually advised students to, first change all external minus signs into plus signs, following the rule

$$–(\pm\, a) = +(\mp\, a),$$

to obtain

$$(+7) + (+3) + (–5),$$

then to remove external plus signs and brackets, and also, more discreetly, the initial predicative sign, which finally gives

$$7 +3 –5,$$

i.e. a mathematically meaningless *sequence of signs and figures* to be really interpreted as the sum

$$7 + 3 – 5.$$

Of course such a strange procedure deserves to be explained in its turn (which I shall not try to do here). But it duly explains why +3 could not be deprived of its plus sign, so far as teachers *could not do without it* in the calculation of algebraic sums.

### *3.3. Pitfalls in curriculum development*

The primary concern of didactic theory is to analyse such sets of conditions and constraints as have appeared in the example just given. Ignorance of didactic constraints leads straight into traps: this is the main point I shall be making from now on.

Curriculum development can be seen as a response to a major, all-pervasive constraint of modern technological society. Now, in going about his business, the curriculum designer meets hosts of constraints. The big problem with any rational-bureaucratic undertaking is that it embodies and, in fact, enacts, a conception of social action as a top-down process. It is the plight of the rational-bureaucratic *ethos* that it is intrinsically governed by a linear, top-down logic, in which every further step is supposed to be made possible by preceding ones: defining desired outcomes; identifying suggested materials; developing suggested materials; and so forth. Such is the curriculum builder's pilgrim's progress through the transparency illusion.

On all levels curriculum developers are likely to be duped. Let us imagine a committee appointed to set up a list of teaching items. Let us suppose more precisely that the committee is trying to define what exactly pupils should be taught about isometries in the Euclidean plane. The committee feels free to choose whatever will be thought consonant with the general educational goals. Let me show now that, in fact, such naive freedom does not exist.

Suppose the committee is a British one. One decision they will have to make is whether, alongside rotations, translations and reflections, pupils should be expected to know something about glide reflections. But if the committee is a French one, the opportunity to do so will simply not exist. Not because glide reflections would be officially prohibited by the French government (which, of course, has not the faintest idea of the matter under discussion). The clue to the riddle is that glide reflections will not be mentioned simply because *they do not exist in the traditional culture of the French mathematics noosphere*. Indeed, French mathematical vocabulary offers no registered name for glide reflections – which, therefore, do not exist even in name.

This phenomenon has its roots in a rather complex situation. Of course glide reflections are recognised as definite mathematical entities by French mathematicians! But, for historical reasons (essentially the old kinematic view of transformations), they have not been acknowledged as "real" transformations (in the plane), much as negative and imaginary numbers were considered "fictitious" numbers. A "real" transformation is in essence a displacement. For that reason, ecological analysis cannot account in the same way for the curricular existence of rotations and translations, on the one hand, and for the case of reflections and glide reflections, on the other. The absence, in the French present-day curriculum, of glide reflections, as well as the particular treatment inflicted on reflections, are two closely linked phenomena.

So much for the mathematical side. Let us now turn to the cultural side. About a quarter of a century ago, a change took place as regards glide reflections, as the international mathematics noosphere became increasingly interested in the theme of tessellations, i.e. two-dimensional crystallography, which received international recognition in connection with M. C. Escher's drawings and paintings. (The theme has recently been superseded by that of chaos.) Now, in two-dimensional crystallography, one cannot evade glide reflections: even French crystallographers knew this, and, of course, had a name for them – "symétrie translatoire", i.e. "translatory reflection", was the name and, if I am well informed, still is –, while French mathematics proudly continued to ignore them. When, two decades ago, small French noospheric circles became interested in tessellations, they chose – well, in fact they did not choose, in ignorance of any available alternative – to translate the wide-spread English name, which was then rendered by "symétrie glissante", or even by "glissage" – a rather unusual and technical word in French (applied only to the sliding down of cut timber). The main point that deserves emphasis in this respect is that two subcultures – the mathematical or, more precisely, the noospheric, and the crystallographical –, linked to two distinct and socially separate social practices, developed almost independantly.

All this relates to a, major constraint of secondary mathematics education. Society tends to demand that any taught mathematical topic be reflected in, and in turn reflect, some of its own cultural interests. The demands of cultural relevance are a very exacting ingredient of the negotiation between society and the teaching system's noosphere. In the case under consideration, for example, the committee members will have to make glide reflections appear as a culturally relevant object of study. They may think that, in looking for "illustrations" and "applications" of glide reflections, they are freely trying to foster the pupils' "motivation" – a typical phenomenon of "false conciousness". But the real motive for their own motivation in looking for cultural embodiments of glide reflections (such as, for

instance, patterns on wallpaper or tiled floors) should be objectively connected with strategic constraints with which they have to cope in the course of the negotiation process.
Ecological analysis thus provides a rather pessimistic, but much more realistic view of the didactic world. In this perspective; curriculum developers are certainly nothing more than factors, among so many others, in the didactic transposition process. They can more aptly be described as transposition "employees". Or, to draw on yet another metaphor, they can be regarded as the amanuenses of didactic transposition. In this respect, the text they will eventually supply teachers with resembles a palimpsest, on which fragments of former texts still show, sometimes to the point that the intended new text will be illegible.

### *3.4. Cultural demands and the noosphere*

Cultural constraints are a very important variety of societal constraints. They usually play a decisive rôle in the formation of curricula. Many facts that can be observed in the classroom are genuine offsprings of cultural constraints. The effects of such constraints can be described in ecological terms of life and death. Some scholastic practices will be promoted, while others may be killed or brutally extinguished.
On the macro-ecological level, the American scholastic scene affords us a good example. The so-called Life adjustment Movement, whose early development took place at the end of the nineteenth century, can be viewed as a bold demand made by the American society on its teaching system. This situation had indeed far-reaching ecological consequences concerning school subject-matters. Some of them were almost reduced to memories, while other, non academic fields of activity began to flourish. After many decades, at a time when this social evolution had hardly taken hold of French society – in which some noospherians advocated the "opening of school onto Life", *l'ouverture* de *l'école sur la vie* –, a highly predictable reaction occurred, known to us as the "Back to Basics" movement, and once again triggered off by society.
It should not be thought, in view of this state of things, that the noosphere was born to be a looser. Indeed, noospherians generally try hard to succeed. But, more often than not, the noospheric activity results in what I call, using the old Freudian jargon, *compromise formations* ("Kompromissbildungen"). To take but one example, mathematicians are well aware that negative numbers are a creation of mathematical activity, more precisely of algebraic activity, and that they cannot be adequately conceived of in terms of so-called "concrete models". Moreover, research in mathematics education has empirically demonstrated that to resort to thermometers and lifts is of very little help for the proper understanding of directed numbers [4]. But the unabashed, ordinary noospherian still goes on his endless Quest for the Holy Grail: in other words, a good, concrete model of directed numbers, including the overworked rule of signs, is still badly "needed".

### *3.5. Teaching and other social practices*

Confronted with the cultural pressure on the teaching system, the noosphere often reacts awkwardly, even wrongly, and noospherians often yield under the pressure of cultural demands to such an extent that, not infrequently, they internalise these demands into an allegedly personal creed. Now the pressure exerted upon them cannot entirely be ascribed to culture. They also have to reckon with yet more demands, which I call *social demands* – a topic conducive to central concepts of didactic transposition theory.
Society is a complex grouping of social practices of very different kinds – from, let us say, mathematics to bricklaying and career couselling. Now suppose that we want to establish

[4] For some remarks on this point, see KUCHEMANN 1981, 86-87; and COQUIN 1985, 182-186.

mathematics, or bricklaying, or career counselling as a school subject-matter. In doing so, we shall define new social practices – the teaching of mathematics, or of bricklaying, or of career couselling. Such a process of establishment will have to satisfy a broad, general *compatibility constraint*, insofar as each newly established "school-bound" social practice – e.g. "teaching and learning mathematics" – should be recognised as a legitimate representative of its "school-free" counterpart – e.g., "doing mathematics". In other words, it should appear to the experts in the field as faithful to the reference social practice, and worthy of the label (e.g., teaching of *mathematics*) to the use of which it lays claim.

Now the distance between the teaching practice and its school-free counterpart usually tends to increase in the course of time and will thereby jeopardise the epistemological (and thus social) legitimacy bestowed on the teaching practice. At interval, the noosphere will have to reconsider the existing curriculum in order to narrow the distance – a vital manipulation which should result in brand-new social legitimacy.

Of course this is part of the negotiation process I referred to earlier. But one should be aware that this negotiation is generally a very comprehensive one, which at the same time will cope with both cultural and social demands.

In this respect, the New Math Movement appears as a valuable case in point. The mathematics noosphere won acceptance for the idea that the change New Math was to bring about would reduce the foreigness of school mathematics to "live" mathematics and, by doing so, would better meet the *needs* of citizens. Unfortunately, the noosphere is generally not able to secure such a position without resorting to compromise formations. Let us take up again the case of the directed numbers.

From the beginning of the nineteenth century onwards, mathematicians had been increasingly concerned with so-called foundation problems. This concern resulted in many developments, including *relative consistency theorems*. It was shown for example that, as long as the theory of *natural* numbers is consistent, the theory of *directed* numbers can be proved to be consistent also. The usual consistency proof relies on the "Weierstrassian trick" of constructing negative numbers as equivalence classes of couples of natural numbers.

Now, in the process of didactic transposition behind the New Math reform, the *proof*, rather than the *theorem*, was selected. For some years, secondary school mathematics teachers learnt to think of integers as sets of couples of natural numbers, while the consistency theorem itself *was essentially ignored*. So that, in passing from mathematicians' mathematics to school mathematics, this piece of (genuine) mathematical knowledge was turned into a rather distorted idea which, for a few, magic years, was imposed upon theories of pupils. In the case in point, the mathematics noosphere, which so often lacks mathematical sensitivity, simply lost its self-control.

What a teaching practice refers to, generally, is not the social practice as such, but a certain *body of knowledge* in which the given social practice is supposed to be embodied. Of course, the word "knowledge" is given here a very wide meaning, including skills and know-how as well as "theoretical" knowledge.

In my book on didactic transposition theory [5], I drew heavily on the notion of *scholarly knowledge* ("savoir savant") .The early scope of the theory was, in fact, limited to mathematics teaching, and mathematics is indeed scholarly knowledge – in a sense I shall now try to clarify.

First of all, the very expression "scholarly knowledge" is in fact what Bourbaki would call "an abuse of language". What may be scholarly is a *body* of knowledge, not knowledge in itself. Scholarliness, in effect, is not an epistemological property; it is essentially a cultural attribute, which, indeed, happens on closer scrutiny to be related to epistemological

[5] CHEVALLARD 1985.

"dispositions" of the body of knowledge in question – a subject I shall leave out here. What is more, there exists a hierarchy of scholarliness, from genuinely scholarly bodies of knowledge to scholarly-like or even pseudo-scholarly ones.
Of course, even non-scholarly bodies of knowledge, such as for example bricklaying, may and can be taught. But their teachability will eventually depend on three main factors, which must in turn be assessed in relation to the social organisation and cultural values. Firstly, their *social value*, or *epistemological relevance* – which essentially accounts for the fact that bricklaying, for example, or even taxidermy, are teachable. Secondly, their *cultural relevance*, which measures their cultural "desirability". Thirdly, the *degree of exposure to society* of both the teaching practice and the corresponding school-free social practice.
Any taught subject-matter has to cope with these three compatibility factors. Noospheres should therefore simultaneously lay claim to the obviously high epistemological relevance of their protégés, try to make them culturally attractive, and, last but not least, conceal them from too much exposure to potential enemies.
To some extent, this applies equally well to any teaching practice. In the context of general education, although it can boast its high cultural *legitimacy*, mathematics teaching lacks cultural *relevance*, and relies essentially on its supposed *epistemological* relevance (its noosphere has for centuries painstakingly argued about its ability to meet "social needs", a strategic term indeed).
In this line, it is worthy of note that noospheric strategies may change, as time goes by, to the point that such strategic moves change the subject-matter contents themselves. When the teaching of mathematics at the secondary level was still *in statu nascendi* – let us say, at the end of the seventeenth century –, its noosphere relied heavily on the fact that mathematics could be made to appear as a wide-embracing discipline, ranging from higher algebra and geometry to stone cutting or even pyrotechnic – a kind of strategy not foreign to that of present-day "informatics", supposed to ensure epistemological relevance and, up to point, cultural relevance.
Let me add that, however, no teaching practice can elude the question of cultural legitimacy. This sometimes leads to amazing curricula, displaying a mixture of both instrumental knowledge and, so to speak, borrowed, ennobling scholarly knowledge. More generally, bodies of knowledge strive for scholarliness, a cultural privilege that can be enjoyed, lost, and recovered, and which constitutes as yet in our societies the best way to cultural legitimacy.

## 4. A foregone conclusion

This is one side of the story. What curriculum developers are left to enjoy now is the much more intricate set of *internal* didactic constraints, i.e. those constraints that are objectively imposed upon the interplay between teacher, student, and taught knowledge – a subject on which Guy Brousseau has provided an illuminating analysis [6].This is the other side of the story. Both sides, however, convey one and the same moral: within the didactic order, nobody is free to do as he or she likes.

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[6] See BROUSSEAU 1986.

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